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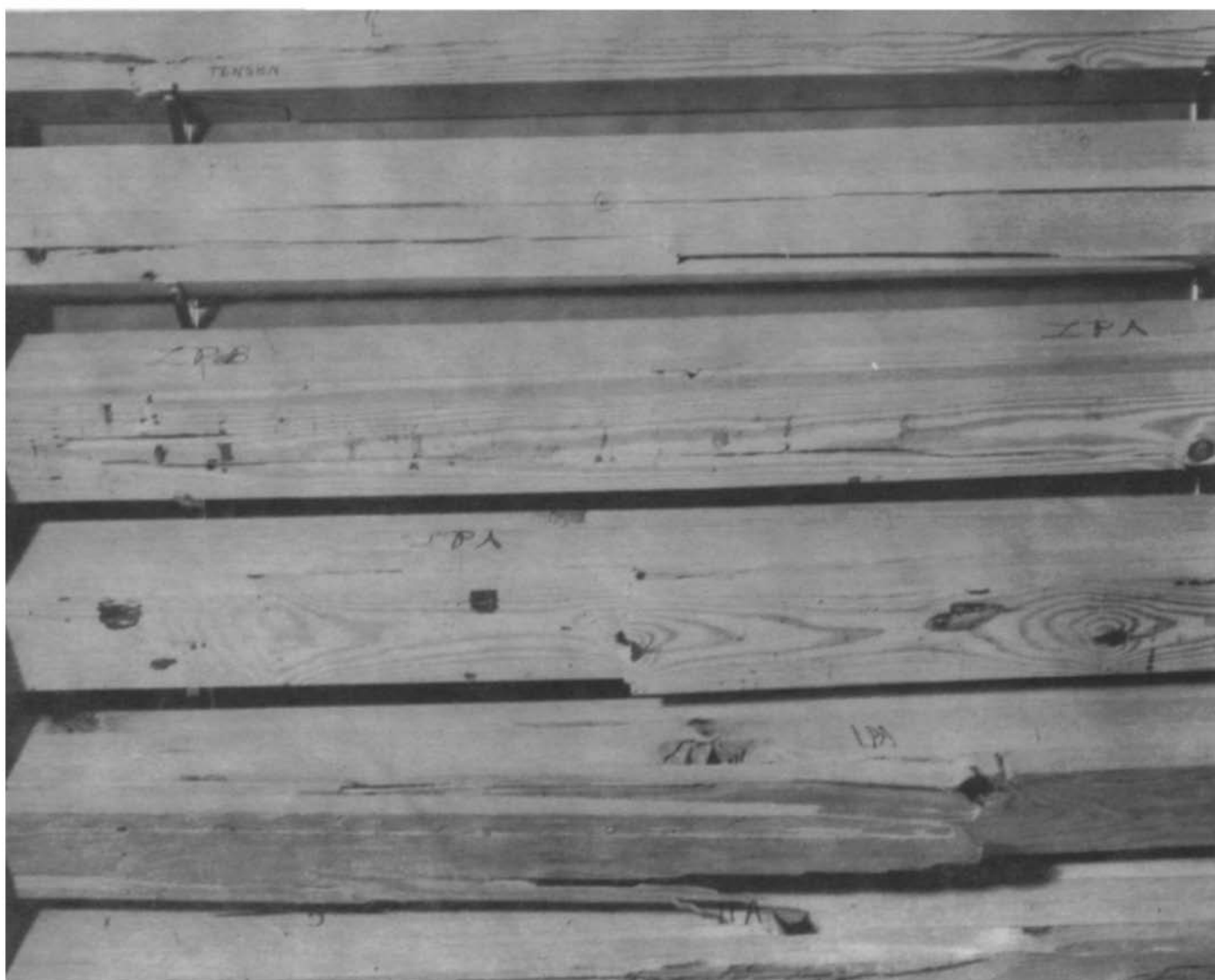
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Bending Strength of Shallow Glued-Laminated Beams of a Uniform Grade



ABSTRACT

Ninety glued-laminated Douglas-fir or southern pine beams of a uniform grade with 2-, 4-, or 6-laminations were evaluated in static bending tests. No specially graded tension laminations or end joints were used.

The purpose of the tests was to determine which of three present design criteria best predict near-minimum bending strength values for shallow glued-laminated (glulam) beams. A variation of a strength ratio concept, with an applied adjustment factor of 0.85, was found to predict the near-minimum strengths more accurately than the I_K/I_G concept now used for deep beams.

Because a new method for determining appropriate design stresses for shallow beams was developed, results will be useful to industry committees establishing specifications.

Bending Strength of Shallow Glued-Laminated Beams of a Uniform Grade²

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INTRODUCTION

Within the past several years, research efforts in glued-laminated timber have concentrated on better defining the strength properties of large glued-laminated beams. Many beams between 12 and 30 inches deep have been evaluated to define required tension lamination grades and to determine the possible benefits of using nondestructively evaluated lumber in their manufacture (13).³

The bending strength of shallow, horizontally laminated beams (12 in. or less in depth) has been determined by design criteria which include either the I_K/I_G concept for deep beams or the strength ratio (SR) concept for single pieces of lumber (defined on page 4). Use of these different methods gave conflicting results and, due to lack of data, it was questionable as to which was the most accurate.

Freas and Selbo (10) presented procedures for determining I_K/I_G values and the corresponding predicted design stresses. The SR approach is based on the principles of the American Society of Testing and Materials (ASTM) D 245 (5). Two variations of this SR method are possible,

depending upon where the maximum-sized knot for a grade is placed. These strength ratios can also be used to predict design stresses.

At the start of this study AITC 117-76 (1) listed design bending strength values for the test beams. Those values were determined by using a combination of the prediction methods, but few data were available to verify them. A study of the strength properties of shallow beams was needed to determine which method was the most accurate for predicting design stresses of shallow beams and to evaluate the reliability of current design values. Such information would permit designers to economically and reliably utilize the lumber resource in the form of glued-laminated timber.

PAST AND CURRENT WORK

Limited research has been conducted on methods to predict design stresses of glulam beams smaller than eight laminations. Five studies containing information that could be useful to this study were located.

Twelve-inch-deep beams were evaluated by Wilson and Cottingham in 1947 (19). The objective of their

research was to find the effect of the size and position of knots on strength and to provide formulas for the design of horizontally laminated beams with knots. Of the 90 Douglas-fir beams tested, 30 had 8 laminations and 60 had 17 laminations. The beams contained knots of different sizes located near the same cross section and at different distances from the neutral axis. It was assumed that the reduction in strength caused by the knots could be measured by I_K/I_G . The tests showed this assumption to be correct with reasonable accuracy, but it was also found that an increase in I_K/I_G was accompanied by an increase in variability. The equation of their suggested design curve is

$$y = (1 + 3x) (1 - x)^3 (1 - x/2)$$

where

$x = I_K/I_G$ and

$y = \text{strength ratio.}$

¹ Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

² Research conducted in cooperation with the American Institute of Timber Construction (AITC).

³ Italicized numbers in parentheses refer to literature cited at end of this report.

USDA Technical Bulletin 1069 (10) recommends this procedure to predict glulam design values in bending.

In 1961 Curry determined working stresses for structural laminated timber (8). Part of Curry's study involved bending tests of shallow beams of horizontally laminated Douglas-fir. These beams were made up of eight 3/4-inch-thick laminations, making them 3 inches wide by 6 inches deep. A limited number of 3-inch-square beams (four laminations) was also tested. As Curry expected, the test results showed a Strong correlation between bending strength and the influence of knots as described by the I_K/I_G concept. Through multiple regression analysis Curry derived equations which related modulus of rupture (MOR) and modulus of elasticity (MOE) to specific gravity and I_K/I_G ratios for each species tested. He then divided each equation by an estimated value for clear material and inserted a nominal value for specific gravity. The result was equations relating strength ratios to I_K/I_G ratios for beams 6 to 75 inches deep.

Pentoney conducted a study on the design criteria for wood laminations stressed in bending in 1963 (15). The two species of lumber tested were coast region Douglas-fir and white oak. Three grades of Douglas-fir were tested: Select Structural, Construction, and Standard. Pentoney presents tables which give recommended strength ratios for Douglas-fir laminations in bending members. These tables are for members with 10 or more laminations. Recommended adjustments for the MOR of horizontally laminated members of less than 10 laminations are also presented in the study. To make adjustments Pentoney assumed that the two outer laminations contained the maximum size knot permissible (one on each face of the beam), while the remaining interior sections contained knots one-third or one-half the maximum size, depending upon the grade of lumber used. All the knots were assumed to be in the same section of the beam. These assumptions are similar to those presented in the minimum SR concept, method B, as discussed later in this report, but they are somewhat less conservative.

Prior to 1969 J. W. Johnson at Oregon State University conducted a study quite similar to the study reported here. Johnson tested 100 uniform grade Douglas-fir beams with 2-, 4-, 6-, or 8-laminations. However,

the results were never completely analyzed or published. With Johnson's permission his results were combined with the results of this study and comparisons are discussed later in this report. The details of Johnson's study and a summary of his test results are presented in appendix A.

Fox (9) reported on the tests of thirty 18-inch-deep, Douglas-fir glulam beams in 1978; however, his report was not available during the planning of this study. The balanced laminating combination that he tested consisted of Canadian laminating grades B, C, and D which are similar to the USA's laminating grades L1, L2, and L3, respectively. Quality of the tension laminations was similar to the L1 grade. The I_K/I_G theory predicted that the selected test beam combination would provide an allowable bending stress of 2,000 pounds per square inch. However, the beams performed far below that level, suggesting that the I_K/I_G theory overestimates the allowable stress level when used as a basis for the derivation of laminating combinations without specially graded tension laminations. Fox found that an allowable stress of 1,500 pounds per square inch might be appropriate for the nominal 20f laminating combination for normal duration of load and dry service conditions. That value is 25 percent lower than the predicted value, suggesting a level for design of deep beams without specially graded tension laminations.

BEAM MATERIALS AND MANUFACTURING

Experimental Design

The experimental design for this study is shown in table 1. Three grade/species were chosen: L1 Douglas-fir, No. 2D southern pine, and L3 Douglas-fir. The three beam sizes chosen were 2-, 4-, and 6-lamination beams. Ten replicates were included in each of the nine beam groups, for a total of 90 test beams.

Table 1.—Experimental design-number of test specimens

Grade and species	Number of laminations		
	2	4	6
L1 Douglas-fir	10	10	10
No. 2D southern pine	10	10	10
L3 Douglas-fir	10	10	10

Lumber Selection and Evaluation

Nominal 2 by 6 lumber 14 feet long was used to manufacture the specimens. A piece of lumber was used only when it contained a representative strength reducing characteristic of the grade/species located within the midlength 7 to 8 feet. This material was selected from the stock on hand at the two laminating plants that manufactured the test beams. The southern pine lumber was grade stamped as No. 2 according to the 1970 Southern Pine Inspection Bureau (SPIB) rules (16) by SPIB supervised mill graders; AITC representatives and a plant grader regraded the lumber as No. 2D at the plant. AITC representatives and a plant grader under West Coast Lumber Inspection Bureau (WCLIB) grading supervisors graded the Douglas-fir material at the plant according to the 1970 WCLIB rules (18).

Each 2 by 6 was randomly selected and then end marked to indicate the sequence number and grouping category. To aid in the analysis of results the moisture content, weight, and MOE were determined for each piece of lumber. The moisture content was determined by averaging three readings taken with a power-loss type moisture meter along the length of each lamination. Both the weight and the MOE value were determined with an E-computer which uses a vibration technique.

The location of each piece of lumber within the beams was recorded, as well as the locations and sizes of all the strength-reducing characteristics in the midlength 7 to 8 feet. Knots were measured on both faces of the laminations and then their averages were recorded. The effective size of all spike knots and those knots not visible on two faces was estimated.

Beam Manufacture

The 90 shallow Douglas-fir and southern pine beams were manufactured during the summer of 1976 by two commercial laminators. All manufacturing conformed to Voluntary Product Standard PS 56-73 for Structural Glued Laminated Timber (17). No end joints were used but, as previously noted, the strength-reducing characteristics of the grades were located near midlength of the

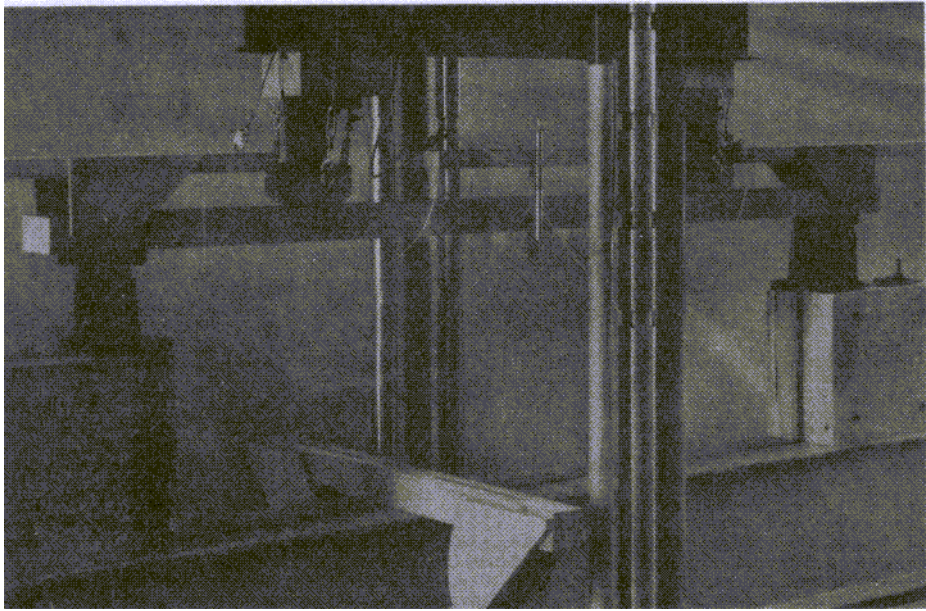


Figure 1.—Floor level view of equipment used to evaluate the 2-lamination beams. Similar, but larger setups were used to test the 4- and 6-lamination beams.

(M 145 296-4)

Table 2.—Assumed lumber properties

Lumber grade	Modulus of elasticity ¹	Clear wood design stress ²	Knot properties ³	
			\bar{X}	h_v
	Million lb/in. ²	Lb/in. ²		
DOUGLAS-FIR				
L1	2.10	3,500	0.069	0.324
L3	1.60	3,000	.116	.464
SOUTHERN PINE				
No. 2D	1.80	3,500	.076	.433

- ¹ From reference (1).
² From reference (4).
³ From reference (12).

beams. Phenol-resorcinol adhesives were used in face gluing the laminations. All 90 beams were surfaced to a 5- 1/8-inch width prior to testing.

RESEARCH METHODS

Test Procedures

The beams were tested according to ASTM D 198 (6). Figure 1 shows the setup for the 2-lamination beams; two-point loading was used. The span between the reactions was 92, 124, and 156 inches for the 2-, 4-, and

g-lamination beams, respectively. Similarly, the distance between the load heads was 50, 40, and 30 inches for the 2-, 4-, and 6-lamination beams, respectively. These dimensions were derived by combining a shear span-to-depth ratio of 14:1 with the intent to have an equal length of each beam subjected to 75 percent or more of the maximum moment. The shear span-to-depth ratio of 14:1 was chosen to maximize the chance of bending type failures, while limiting the probability of failure due to horizontal shear. The tension side of

each beam was randomly selected.

A small load was applied to the test beams to assure proper alignment of gages and equipment before they were continuously loaded to failure. The test machine head movement was continued until the load dropped to about 50 percent of the maximum load. Machine head speeds were such that the maximum load was reached in the time specified by ASTM D 198 (6).

Beam Preparation

All of the test specimens were manufactured from 14-foot material; the 6-lamination beams were tested full length, but the 2- and 4-lamination beams were cut to lengths of 104 and 136 inches, respectively. So that the known strength-reducing characteristics would be subjected to the maximum bending moment during testing, the center of each beam was located and equal lengths cut from each end.

Data Obtained

Just prior to testing, the beams were measured, marked, and weighed. Lines were drawn and then labeled at the centerline and the two load points so that the area of beam failure could be easily located. Cross-sectional dimensions at the load points were recorded as well as the total length of each beam.

A continuous record of the machine test load versus the full span deflection was obtained during the test with an X-Y recorder. Yoke deflectometers were used to support the linear variable differential transducer (LVDT) which measured the full-span deflection. This type of setup recorded the desired data up to failure with no threat of damage to the equipment. Details of the failures and the probable initiation points were also noted during the test.

Predicted Design Stresses

Lumber properties for the three grade/species studied are given in table 2. The MOE values were obtained from AITC 117- 76 (1) the clear wood design stress values from ASTM D 3737-78 (4), and the knot properties from Moody (72). The I_k/IG concept requires use of the MOE values and the knot properties.

Table 3.—Predicted design stress values for 2-, 4-, 6-, or 8-lamination beams

Species and grade	Number of laminations	Strength ratio			Predicted design stress ³		
		I _K /I _G concept ¹	Minimum SR concept ²		I _K /I _G concept	Minimum SR concept	
			Method A	Method B		Method A	Method B
					Lb/in. ²	Lb/in. ²	Lb/in. ²
L1 Douglas-fir	2	.557	.75	.562	1,950	2,630	1,970
	4	.606	.75	.683	2,120	2,630	2,390
	6	.662	.75	.710	2,320	2,630	2,490
	8	.701	.75	.722	2,450	2,630	2,530
No. 2D southern pine	2	.401	.57	.434	1,400	2,000	1,520
	4	.462	.57	.480	1,620	2,000	1,680
	6	.535	.57	.480	1,870	2,000	1,680
L3 Douglas-fir	2	.312	.50	.250	940	1,500	750
	4	.370	.50	.417	1,110	1,500	1,250
	6	.443	.50	.450	1,330	1,500	1,350
	8	.496	.50	.464	1,490	1,500	1,390

¹ Based on procedures given in USDA Technical Bulletin 1069 (10) and ASTM D 3737 (4) also based on knot data in (12).

² Based on procedures in ASTM D 245 (5) and described in detail in this report under "Predicted Design Stresses."

³ Predicted design stress for a uniformly loaded beam with a 21:1 span-to-depth ratio and a 12 pct moisture content.

⁴ Placing the knots along the edge of the laminations furthest from the neutral axis is no longer the worst position because of the different maximum sizes of edge and center knots allowed for No. 2D southern pine. Stacked centerline knots were used to produce the lowest strength ratios.

The predicted design stresses in table 3 were obtained by multiplying the strength ratios (also in table 3) by the appropriate clear wood design stresses in table 2. The strength ratios were calculated using the I_K/I_G concept or the minimum strength ratio concept, methods A or B. Those three prediction methods are explained in more detail below.

I_K/I_G

The I_K/I_G concept, based on the principles given in USDA Technical Bulletin No. 1069 (10), is one means of estimating the strength reduction caused by knots. This bulletin gives a design curve which relates strength ratios to I_K/I_G ratios. (I_K is the sum of the moments of inertia of the cross-sectional areas of all knots within 6 inches of a critical cross section and I_G is the moment of inertia of the full cross section.) The I_K/I_G concept, therefore, indirectly predicts a design stress by an empirical relationship. Because it is impractical to determine the actual I_K/I_G ratio of each beam, I_K/I_G values which are likely to be exceeded only infrequently were estimated from the results of statistical knot distribution surveys.

Strength Ratio

Two variations of the ASTM D 245 (5) method were considered and for this study are given the titles of minimum SR concept, methods A and B. Both methods directly predict a

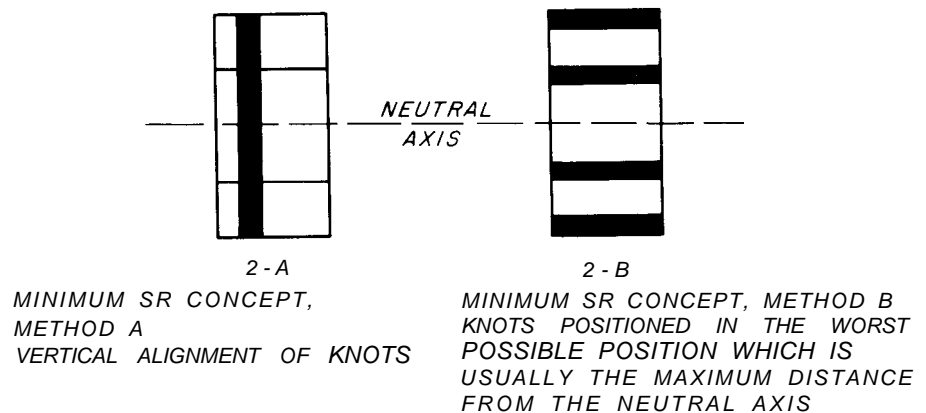


Figure 2.—Two methods to account for the reduction in section modulus due to knots.

(M 148 623)

design stress based upon the reduction of the section modulus due to knots.

Method A strength ratios were determined by vertically aligning the maximum permissible size knot (16, 18) in each lamination (fig. 2A). The L1 and L3 Douglas-fir grades have maximum allowable sized knots which are independent of the location of the knot across the width of the lumber. However, the No. 2D southern pine grade has a different maximum allowable sized knot, depending upon whether the knot is located on the edge or in the center of a piece of lumber. The average of the maximum

permissible edge and centerline knots was used to predict the design stresses for the No. 2D southern pine beams. Use of just the maximum centerline knot, as well as of just the maximum edge knot was examined, but, as will be discussed later, the predicted values using either of those knots did not fit in as well with the Douglas-fir data.

Method B is similar to method A except that the maximum permissible size knot is placed in the worst possible position, usually along the edge of each lamination farthest from the neutral axis (fig. 2B).

Analysis Procedures

Adjustment Factors Applied to MOE Values

The MOE values were adjusted to a 12 percent moisture content following ASTM D 2915 (3). The adjustment of the MOE data for depth and loading conditions was negligible.

Adjustment Factors Applied to MOR Values

Several adjustment factors were required for the MOR values before comparisons could be made with J. W. Johnson's unpublished data, predicted design values, and AITC design values (1). The applied adjustment factors for both MOE and MOR are listed in appendix B, table 8.

(1) Adjustments applied to MOR data for comparison with Johnson's data and predicted design stresses.—Both Johnson's MOR data and the MOR data from this study were adjusted to standard conditions. These conditions imply a 12 percent moisture content and a 12-inch-deep beam with a uniform load and a 21:1 span-to-depth ratio. The moisture content adjustments were determined from ASTM D 2915 (3). Just one factor

for each beam size (7) accounts for the rest of the adjustments to standard conditions (appendix B). Values adjusted in this manner were also used for comparison with predicted design stresses.

(2) Adjustments applied to MOR data for comparison with AITC design values.—Different adjustments were required for comparison with design values. No adjustment for depth was necessary because the design values given in (1, 2) apply to beams 12 inches or less in depth. The published glulam beam design values also imply conditions of uniform loading, a 21:1 span-to-depth ratio, and a 12 percent average moisture content. The adjustments to uniform loading and a 21:1 span-to-depth ratio for the 4- and 6-lamination beams were neglected because they were less than 3 percent (7), but the 2-lamination beam adjustment of 0.925 (7) was used. The moisture content adjustments again followed ASTM D 2915 (3).

Calculation of Near-minimum Values

Estimated near-minimum bending strength values are needed before the test results can be compared with the

AITC design values or the procedures used to predict those design values. The type of statistical distribution for the population must be assumed before a near-minimum value can be calculated from a set of data. A sample size of 10 is inadequate to determine the true type of distribution, thus several analyses of variance (11) were conducted to determine if any of the data could be combined to provide a larger sample size. The analysis of variance, described in more detail in appendix C, showed that the number of laminations did not have a significant effect on the MOR with 95 percent probability; thus the three sizes of beams were combined for some of the analysis.

Near-minimum bending strength values were calculated assuming a lognormal distribution; a 75 percent confidence level at the fifth percentile was chosen. That distribution and confidence level has been used previously to calculate near-minimum values for glulam beams. The necessary statistical factors were found in the appropriate confidence/tolerance table (14) and are given in appendix B, table 9.

The calculated near-minimum values were divided by the 2.1 factor

Table 4.—Summary of test results¹

Number laminations	Specific gravity	Modulus of rupture						Modulus of elasticity			
		Unadjusted			Adjusted to standard conditions ²			Unadjusted		Adjusted to 12 percent moisture content ³	
		Mean	Range	Coefficient of variation	Mean	Range	Coefficient of variation	Mean	Coefficient of variation	Mean	Coefficient of variation
		Lb/in. ²	Lb/in. ²	Pct	Lb/in. ²	Lb/in. ²	Pct	Million Lb/in. ²	Pct	Million Lb/in. ²	Pct
L1 DOUGLAS-FIR											
2	0.51	7,930	5,530- 9,630	17.5	6,670	4,690-8,090	17.4	2.31	11.2	2.16	11.5
4	.50	8,640	7,120-10,660	14.9	7,580	6,160-9,350	15.6	2.18	8.7	2.05	8.9
6	.51	7,890	4,920-10,520	18.5	7,070	4,380-9,300	18.5	2.34	6.9	2.23	6.9
NO. 2D SOUTHERN PINE											
2	.55	6,590	4,640- 8,840	22.9	5,850	4,040-7,940	23.4	1.69	14.7	1.65	14.8
4	.55	6,040	4,500- 7,690	18.8	5,520	4,160-6,970	18.8	1.78	8.2	1.73	8.1
6	.55	5,880	3,550- 9,030	28.7	5,510	3,300-8,550	28.8	1.73	10.3	1.69	10.2
L3 DOUGLAS-FIR											
2	.52	4,870	2,790- 6,450	22.4	4,080	2,310-5,420	22.1	1.86	7.9	1.74	8.1
4	.50	4,410	2,400- 6,710	35.4	3,800	2,050-5,780	35.5	1.79	12.5	1.67	12.4
6	.49	4,220	2,920- 5,630	22.1	3,710	2,580-5,020	22.0	1.75	6.6	1.64	6.5

¹ Each value is the result of 10 tested beams, except for the L3 2-lamination MOE means which are the result of 9 tested beams.

² Adjusted to standard conditions which are a 12 pct moisture content (3) and a 12-in.-deep beam, uniformly loaded with a 21:1 span-to-depth ratio (7).

³ Adjusted to 12 pct moisture content only (3).



Figure 3.—Near-maximum sized knots permitted in L1 Douglas-fir, No. 2D southern pine, and L3 Douglas-fir.

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that has been widely used in the lumber industry to reduce test data from a near-minimum stress level to a bending design stress level. Referred to as “test values” in this report, these adjusted near-minimum values can be compared to AITC design values.

PRESENTATION AND DISCUSSION OF RESULTS

A summary of the test results is presented in table 4. Each tabulated value is the average of 10 tested beams except for the L3 2-lamination MOE means. Those values are the averages of only nine beams because the load versus deflection plot was not obtained for one beam.

The Douglas-fir beams had an average moisture content of about 8 percent and the southern pine beams had an average moisture content of about 10 percent. The MOR values at test conditions (unadjusted)

and those adjusted to standard conditions are both given in table 4. One set of MOE values given in that table are unadjusted while the other has been adjusted to a 12 percent moisture content.

Test Failures

The majority of the test beams failed in the tension lamination at a knot or the grain deviation associated with a knot. This pattern was expected because every lamination selected for the tests had a strength reducing characteristic typical of that particular grade. Figure 3 shows examples of the near-maximum sized knots permitted in the three grades of beams tested. Some of the high- and low-strength beam failures are shown in figures 4 through 7.

About 20 percent of the beams exhibited some form of compression failure prior to rupture of the tension laminations. Compression wrinkles occurred most frequently in the higher strength L1 Douglas-fir grade

and least frequently in the lower strength L3 Douglas-fir grade. In general, beams with compression failures were among the higher strength beams in their beam groups.

Exceptions to the general tension or compression type failures did occur. One 4-lamination southern pine beam fractured through what appeared to be a preexisting compression failure in the outer tension lamination, while another beam broke at what appeared to be a poor glue bond between two of the laminations; both of these beams had near average strengths. As could be expected, eight other beams showed evidence of poor glue bonding in the regions of large knots and steep grain deviation; most of those beams were near average strength No. 2D southern pine beams, but two of them were the lowest strength beams in their beam groups and one beam (shown in the bottom of figure 5) was the highest strength beam in its group.

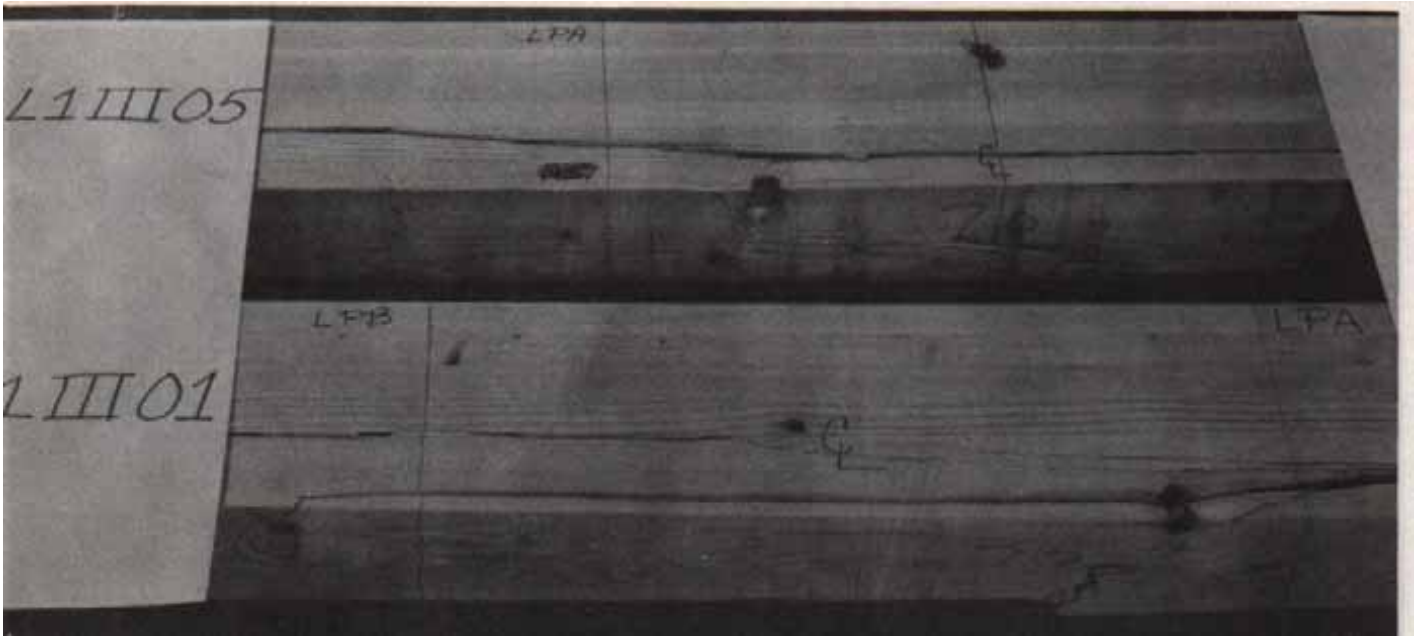


Figure 4.—Failure portions of low-strength 6-lamination L1 Douglas-fir beams.

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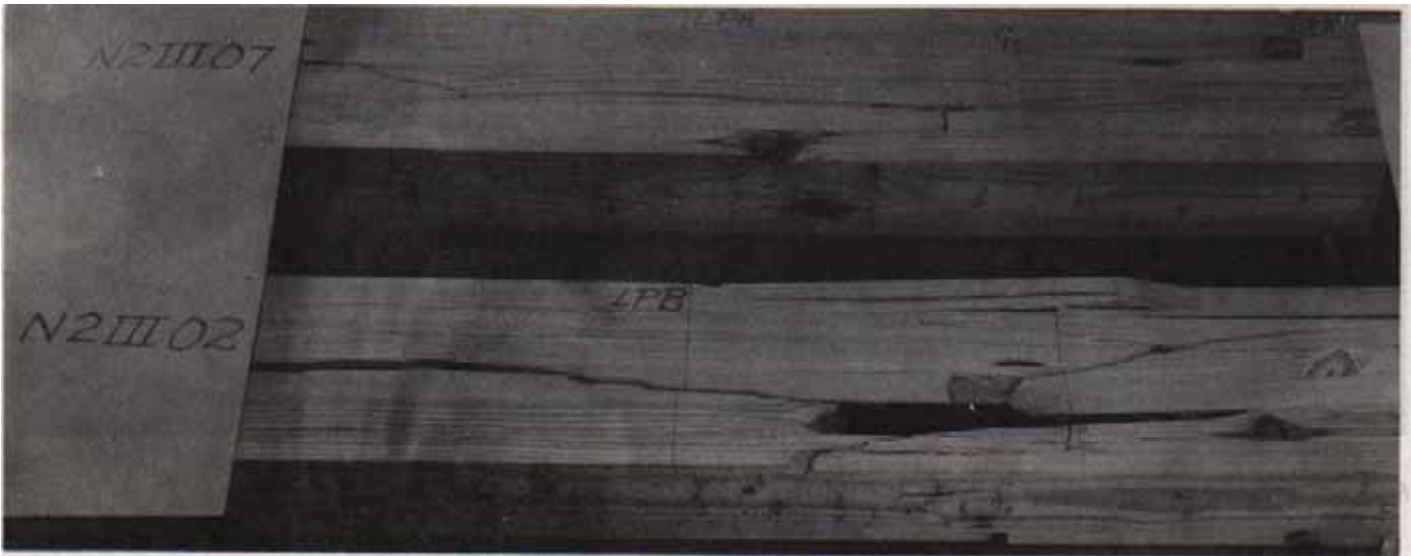


Figure 5.—Failure portions of high-strength 6-lamination No. 2D southern pine beams.

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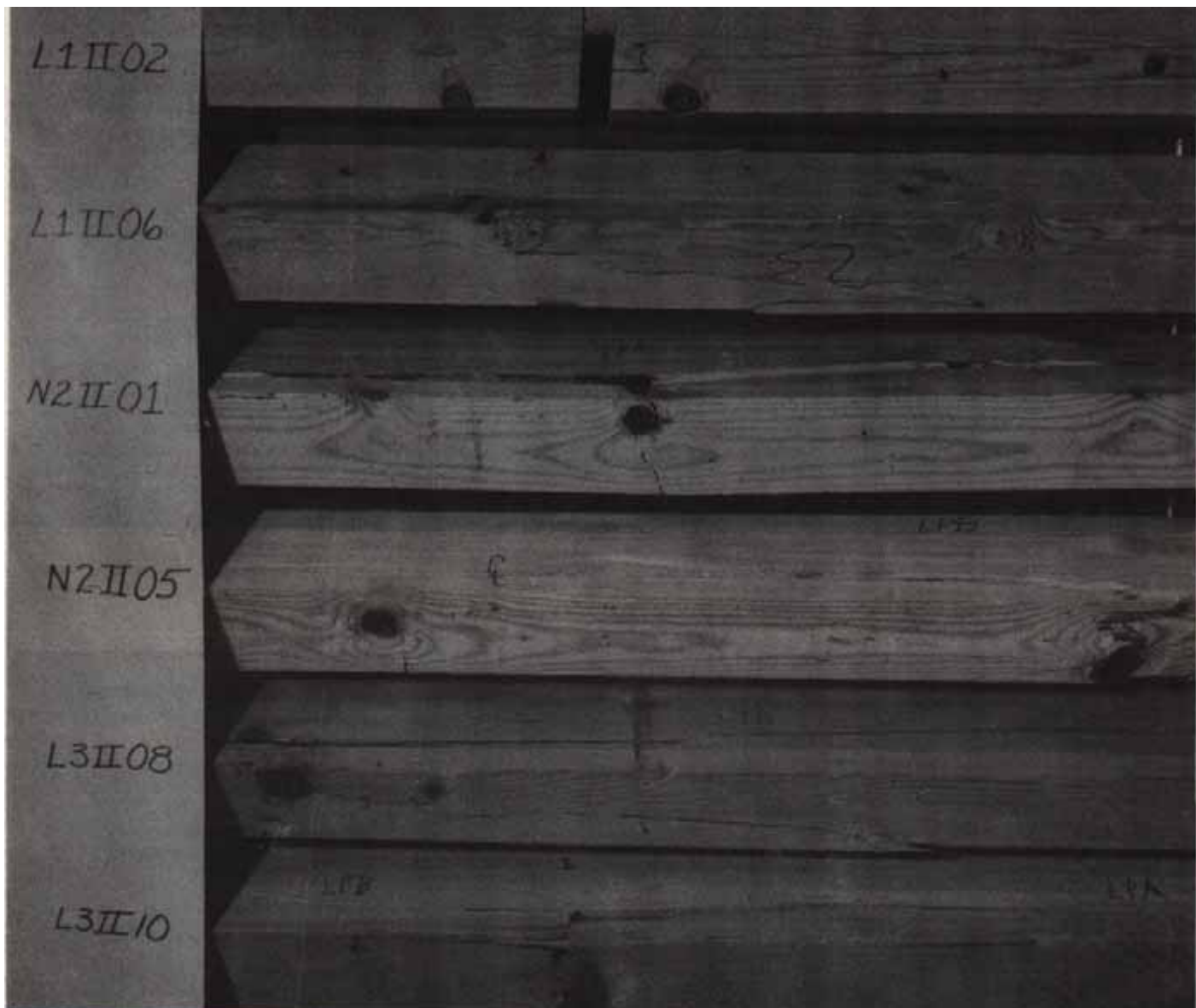


Figure 6.—Failure portions of low-strength 4-lamination beams.

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Comparison with Johnson's MOR Data

Figures 8 through 11 show the individual MOR values from the two studies. When compared with the individual values from this study, J. W. Johnson's unpublished values appear to be slightly higher. An analysis of variance, however, revealed that Johnson's beams with knots occupying 0.1 and 0.2 of the cross section were not significantly different than this study's L1 beams. Similarly,

Johnson's beams with knots occupying 0.4 and 0.5 of the cross section were not significantly different than this study's L3 beams. (See appendix C for a more detailed explanation of the analysis of variance results.)

Comparison with Predicted Values

The MOR test values were compared with bending strength values predicted by the three different methods; the test MOE values were

compared with those predicted by a transformed section analysis.

MOR Test Data

In figures 8 through 11 individual MOR test data adjusted to standard conditions (3, 7) can be compared with predicted design stresses times 2.1.

Figure 12 shows the design stresses (times 2.1) predicted by the I_k/I_G concept and the minimum SR concept, methods A and B. Also

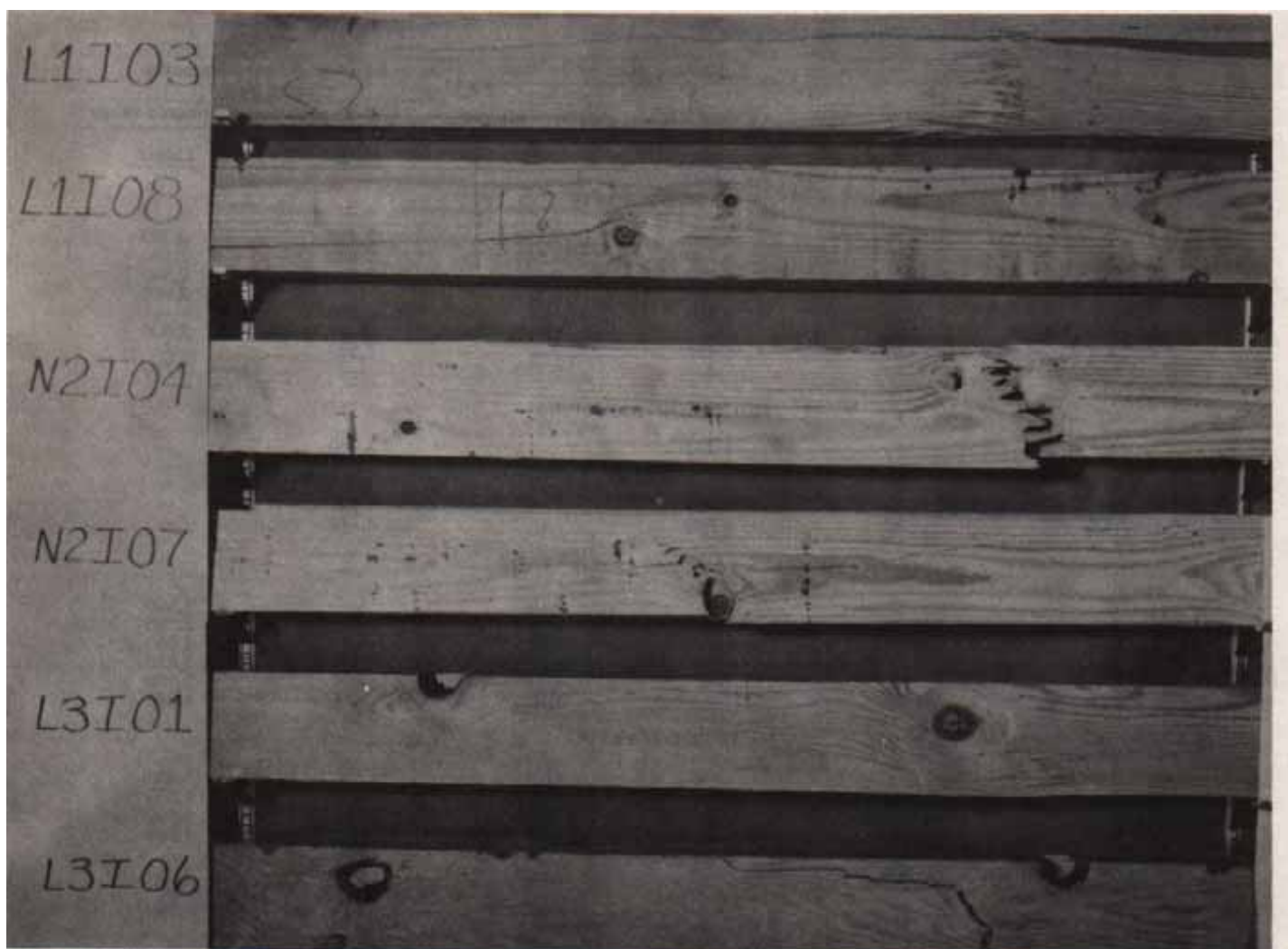


Figure 7.—Failure portions of high-strength 2-lamination beams.

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shown in figure 12 are the actual test minimums and the estimated near-minimums from this study. (The actual test minimum is the lowest MOR value for each beam group, while the estimated near-minimum for each beam group was calculated by assuming a lognormal distribution as previously discussed.)

Many observations can be made by examining figure 12. One such observation is that method A considerably overestimated most of the near-minimum values. Both the I_K/I_G and method B concepts also overestimated a few of the near-minimum values; no difference could be detected between those two methods because none of the near-minimum

values from this study fell between their predicted values. No general increase in bending strength could be observed from the 2- to 4- to 6-lamination beams, as both the I_K/I_G and method B concepts predict.

A comparison can also be made between the predicted values and Johnson's Douglas-fir data. Those actual test minimums and near-minimums estimated with the lognormal distribution are shown in figure 13. (Near-minimums were not calculated for L2 because there was only a sample size of five to work with.) Johnson's results seem to confirm the results from this study. Once again there does not appear to be a definite trend of an increase in bend-

ing strength with an increase in the number of laminations; this absence of a trend indicates that the I_K/I_G and method B concepts may not be very good prediction methods for these shallow beams. However, the method A concept predicted values that were too high; it overestimated one-half of the near-minimums shown in figure 13.

The results from both studies agree with those found by Fox (9) and suggest that the present prediction methods overestimate the strength of glulam beams having conventional lumber grades as tension laminations. Therefore, a more realistic design stress for shallow glulam beams without specially graded ten-

Table 5.—Estimated near-minimum bending strengths

Number of laminations	Number of beams	Estimated near-minimums ¹		Estimated near-minimums divided by 2.1 ²	
		This study	Johnson's study	This Study	Johnson's study
		Lb/in. ²	Lb/in. ²	Lb/in. ²	Lb/in. ²
L1 DOUGLAS-FIR					
2	10	5,200	7,120	2,480	3,390
4	10	5,710	4,500	2,720	2,140
6	10	4,720	5,440	2,250	2,590
8	10	—	6,220	—	2,960
4,6	20	5,320	—	2,530	—
4,6,8	30	—	5,450	—	2,600
2,4,6	30	5,430	—	2,590	—
2,4,6,8	40	—	5,720	—	2,720
NO. 2D SOUTHERN PINE					
2	10	4,090	—	1,950,	—
4	10	3,820	—	1,820	—
6	10	2,940	—	1,400	—
4,6	20	3,490	—	1,660	—
2,4,6	30	3,710	—	1,770	—
L2 DOUGLAS-FIR					
2	5	—	3,190	—	1,520
4	5	—	4,540	—	2,160
6	5	—	3,460	—	1,650
8	5	—	3,420	—	1,630
4,6,8	15	—	4,200	—	2,000
2,4,6,8	20	—	3,960	—	1,890
L3 DOUGLAS-FIR					
2	10	2,790	3,050	1,330	1,450
4	10	1,750	2,840	830	1,350
6	10	2,320	3,280	1,100	1,560
8	10	—	3,230	—	1,540
4,6	20	2,120	—	1,010	—
4,6,8	30	—	3,280	—	1,560
2,4,6	30	2,330	—	1,110	—
2,4,6,8	40	—	3,300	—	1,570

¹Calculated by assuming a lognormal distribution with 75 pct confidence at the fifth percentile. The values given have been adjusted for moisture content. No depth adjustment was required because design values apply to beams 12 in. or less in depth. The method of loading and span-to-depth ratio adjustments for the 4-, 6-, and 8-lamination beams were determined to be less than 3 pct and were neglected. The a-lamination beam values for Johnson's study and this study, however, were divided by their calculated adjustments of 0.941 and 0.925, respectively (7).

²Dividing by 2.1 results in a value which can be compared with AITC design values in table 6.

sion laminations may be predicted by using method A with an applied adjustment factor. A later section in this report further develops this new prediction procedure and gives the necessary adjustment factor.

MOE Test Data

A transformed MOE for each beam tested was determined by taking the MOE values obtained from the E-computer for each piece of lumber in the beam and then applying a transformed section analysis. Figure 14 compares the actual test MOE obtained from the load versus deflection plot with the transformed MOE for each test beam. A regression analysis suggested a line of best fit as

$$Y = 0.955 X + 0.066 \quad (1)$$

where

Y = the actual MOE (million lb/in.²)

X = the transformed MOE (million lb/in.²)

The coefficient of determination (R^2) was 0.92. Overall, the actual MOE values averaged 98.9 percent of the transformed MOE values, suggesting an equation of the form

$$Y = 0.989X \quad (2)$$

where factors are as previously described.

This is slightly higher than previous results (13) and the 0.95 factor currently being used along with assumed lumber MOE values (such as those given in table 2) to predict beam MOE values.

Comparison with AITC Design Values

Comparisons are made with the design values published in AITC 117-76 (1) and AITC 117-79 (2). (Note: because the 117-79 bending strength design values were determined using the new prediction method developed in this report, it is expected that those design values will appear more reasonable than the 117-76 values.)

Table 6.—Comparison of this study's test values with AITC design values¹

Modulus of rupture				Modulus of elasticity		
4- and 6-lamination beams		2-, 4-, and 6-lamination beams		Average design values		Average of test beams ⁴ (unadjusted)
AITC design values	Test	AITC design 117-79	Test values ³	AITC 117-76	AITC 117-79	
Lb/in. ²	Lb/in. ²	Lb/in. ²	Lb/in. ²	Million Lb/in. ²	Million Lb/in. ²	Million Lb/in. ²
L1 DOUGLAS-FIR						
2,600	2,530	2,200	2,590	2.1	2.0	2.27
NO. 2D SOUTHERN PINE						
2,100	1,660	1,600	1,770	1.8	1.7	1.73
L3 DOUGLAS-FIR						
1,200	1,010	1,250	1,110	1.6	1.5	1.80

¹ Test values are near-minimums divided by 2.1 and are from table 5.

² Based on 20 tests.

³ Based on 30 tests.

⁴ Each MOE value given is the average of 30 2-, 4-, and 6-lamination beams. The Douglas-fir beams had an average moisture content of about 8 pct and the southern pine beams had an average moisture content of about 10 pct.

MOR Test Values

As mentioned earlier, test values in this report are defined as estimated near-minimum values divided by 2.1, a factor used to adjust from a near-minimum level to a bending design stress level. Those values can be compared with design values and are given in tables 5 and 6. The test values listed in columns 2 and 4 of table 6 were adjusted as previously discussed.

The 4- and 6-lamination beams in column 2 can be compared with the AITC 117-76 values in column 1 which apply to shallow beams containing 4 or more laminations. The test values were all lower than the 117-76 design values; the L1 Douglas-fir values by less than 3 percent, the No. 2D southern pine values by 21 percent, and the L3 Douglas-fir values by 16 percent. When compared to Johnson's 4-, 6-, and 8-lamination test values in table 5, however, the 117-76 design values appear more reasonable. J. W. Johnson's L1 test value of 2,600 pounds per square inch is the same as the L1 design value; his L3 test value of 1,560 pounds per square inch is 30 percent higher than the L3 design value. Also, Johnson's 4-, 6-, and 8-lamination data with knots occupying 0.3 of the cross section, assumed to be L2 Douglas-fir, resulted in a 2,000 pounds per square inch test value which is 11 percent

higher than the AITC 117-76 design value of 1,800 pounds per square inch.

The 2-, 4-, and 6-lamination test values in column 4 of table 6 can be compared with the AITC 117-79 design values in column 3, which now apply to shallow beams with two or more laminations. As expected, the 117-79 design values appear more reasonable. The 117-79 design values for L1 Douglas-fir and No. 2D southern pine are conservative when compared with the test values. This study's L3 Douglas-fir test value is lower than the 1,250 pounds per square inch design value, but Johnson's L3 Douglas-fir value of 1,570 pounds per square inch is well above that design level. Thus, when data from both studies are considered, the 1,250 pounds per square inch design value for L3 Douglas-fir appears reasonable. Johnson's L1 and L2 2-, 4-, 6-, and 8- lamination Douglas-fir test values of 2,720 and 1,890 pounds per square inch, respectively, are also greater than their corresponding design values; that L2 test value is 11 percent higher than the 1,700 pounds per square inch design value.

MOE Test Values

As shown in table 6, the average MOE values for the two grades of Douglas-fir tested are both greater than the previous and current AITC

design values (1, 2). The L1 and L3 Douglas-fir values are closer to the AITC 117-76 design values than the 117-79 design values, however. So are Johnson's values of 2.15 and 1.78 million pounds per square inch for L1 and L3 Douglas-fir, respectively. Similarly, the L2 Douglas-fir average value of 1.99 million pounds per square inch is closer to the AITC 117-76 design value of 1.8 million pounds per square inch than to the 117-79 design value of 1.7 million pounds per square inch.

The average No. 2D southern pine MOE value fell between the MOE values listed in AITC 117-76 and 117-79, but was closer to the 117-79 value of 1.7 million pounds per square inch.

Development of a New Method for Determining Design Stresses

As previously discussed, the test data indicate that more reliable design stresses may be predicted with the SR concept, method A, if an adjustment factor is applied. To obtain a best estimate of the adjustment factor, the data from this study and Johnson's study were combined for a total of 190 shallow beams. Previous analyses of variance had revealed significant effects on bending strength due to grade, but not the number of laminations (see ap-

pendix C). In an effort to remove that grade effect, the data were normalized by dividing each individual MOR (adjusted to a 12 pct moisture content only) by a value equal to the clear wood design stress times both 2.1 and a SR. Each SR was equal to 1.00 minus the appropriate knot size expressed as a fraction of the lumber width prior to laminating. The knot sizes in Johnson's study were 0.1, 0.2, 0.3, 0.4, and 0.5. The knot sizes in this study were assumed to be equal to the maximum allowable knot sizes. Those knot sizes are 0.25 for L1 Douglas-fir and 0.50 for L3 Douglas-fir, regardless of the location of the knots. The average of the maximum allowable edge knot and centerline knot sizes for nominal 2 by 6 lumber, 0.43, was chosen to calculate the No. 2D southern pine SR. (This average seemed to fit in with the Douglas-fir results better and is further explained in appendix C.)

An analysis of variance was conducted with the 190 bending strength values normalized as mentioned. With this normalized data, neither the grade nor the number of laminations was found to have a significant effect on MOR. That finding indicates the method of normalization used was effective in removing the previous grade effect.

Adjustment factors of 0.86 and 0.92 were calculated by respectively assuming a normal and lognormal distribution with 75 percent confidence at the fifth percentile. The best estimate of the adjustment factor using a nonparametric technique resulted in a factor of 0.85. This 0.85 factor is believed to be the best estimate of the true adjustment factor.

The results of this study and Johnson's study, therefore, suggest that the following equation be used to determine design bending strength values for shallow, visually graded, glulam beams without specially graded tension laminations:

$$F_{bxx} = CWDS \times SR \times 0.85$$

where

F_{bxx} = design value for bending about X-X axis (load applied perpendicular to the wide face)

CWDS = clear wood design stress ((4))

SR = strength ratio = 1.00 minus maximum allowable knot size expressed as a fraction of the lumber width prior to laminating⁴

and

0.85 = adjustment factor based on 190 2- to 8-lamination beams from this study and Johnson's study.

Using this equation, we are 75 percent confident that 95 percent (19 out of 20) of the near-minimum test data from short-term tests of shallow glulam beams will exceed $2.1 \times F_{bxx}$.

CONCLUSIONS

Bending tests of Douglas-fir and southern pine glulam beams of 2-, 4-, or 6-laminations revealed the following about the accuracy of the three methods for predicting near-minimum bending strengths:

1. The minimum SR concept, method A, overestimated most of the near-minimum values.
2. No difference could be detected between the minimum SR concept, method B, and the I_K/I_G concept. These two methods also overestimated some of the near-minimum values. In addition, the data revealed no general trend of increase in the bending strength values from 2-to 4-to 6- laminations as both the I_K/I_G and method B concepts predict.
3. A new prediction method was developed in this report which suggests that better estimates of design bending strength values may be obtained by using the SR concept, method A, with an applied adjustment factor of 0.85. These results agree with the results of both J. W. Johnson (unpublished) and Fox (9), again suggesting that present prediction methods overestimate the strength of glulam beams without specially graded tension laminations.

The Douglas-fir MOE data from this study and Johnson's study agree more closely with the MOE design values in AITC 117-76 than with the lower MOE design values in 117-79. The No. 2D southern pine MOE data from this study, however, agree more

closely with the MOE design values AITC 117-79 than with the higher MOE design values in 117-76. Average beam MOE values were higher than predicted MOE values calculated using the current 0.95 factor with a transformed section approach.

⁴ These criteria may not be generally applicable to structural grades of lumber having different allowable edge and centerline knots. For the No. 2 southern pine grade in this study the average of the maximum allowable edge and centerline knot sizes appeared most appropriate to use in calculating the SR. Preliminary analysis of data collected for a subsequent study suggested that just the maximum allowable edge knot size may be appropriate to use in calculating the SR for the No. 1 southern pine grade.

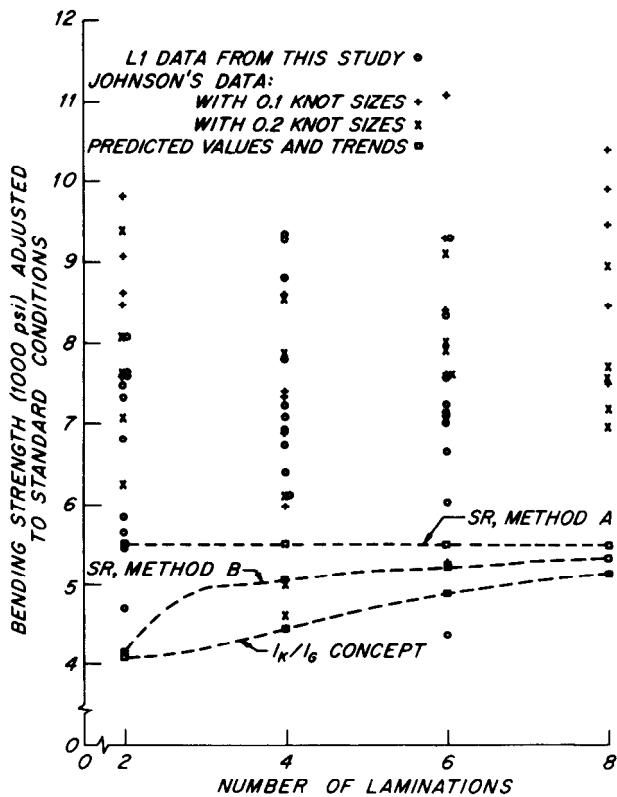


Figure 8.—Individual L1 Douglas-fir MOR values from both this study and Johnson's study are shown along with predicted values and trends. (M 148 624)

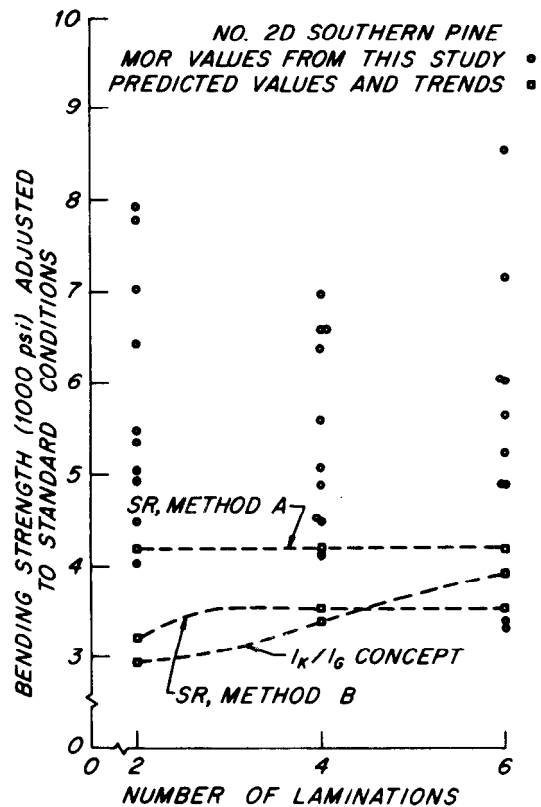


Figure 9.—Individual No. 2D southern pine MOR values from this study are shown along with predicted values and trends. (M 148 625)

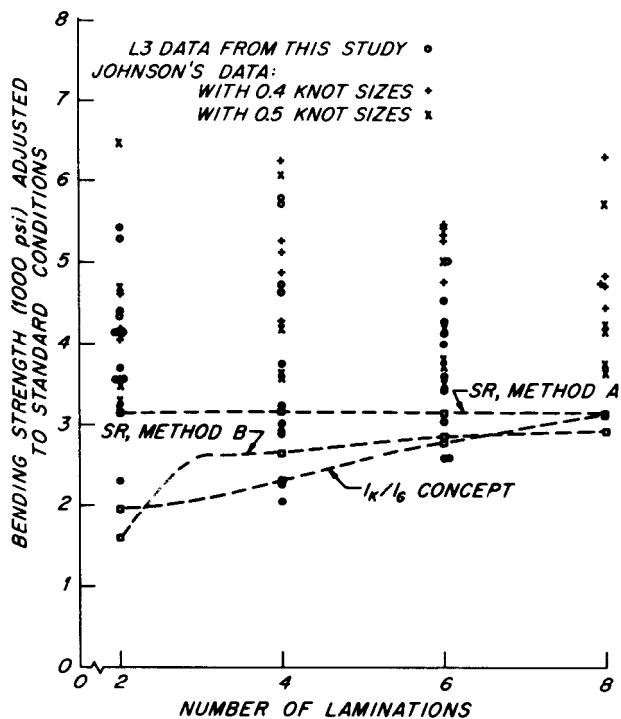


Figure 10.—individual L3 Douglas-fir MOR values from both this study and Johnson's study are shown along with predicted values and trends. (M 148 626)

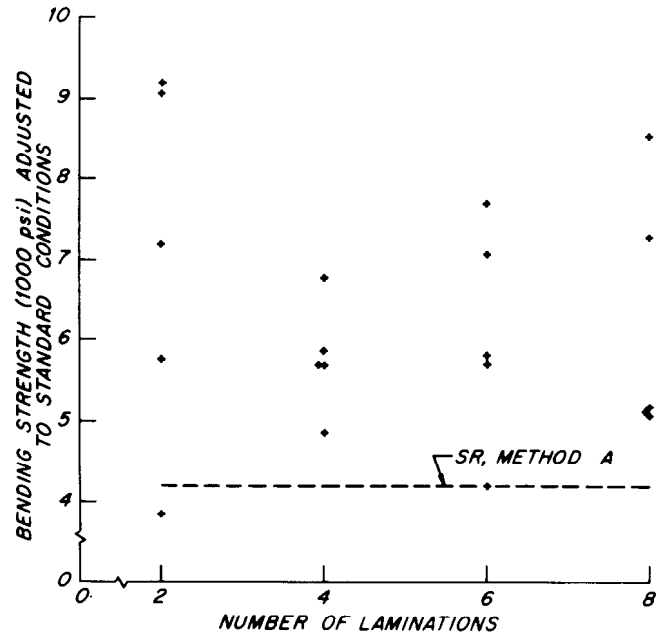


Figure 11.—Individual MOR values for Johnson's beams with knots occupying 0.3 of the cross section are shown along with the predicted values and trends using just one of the three prediction methods. (M 148 627)

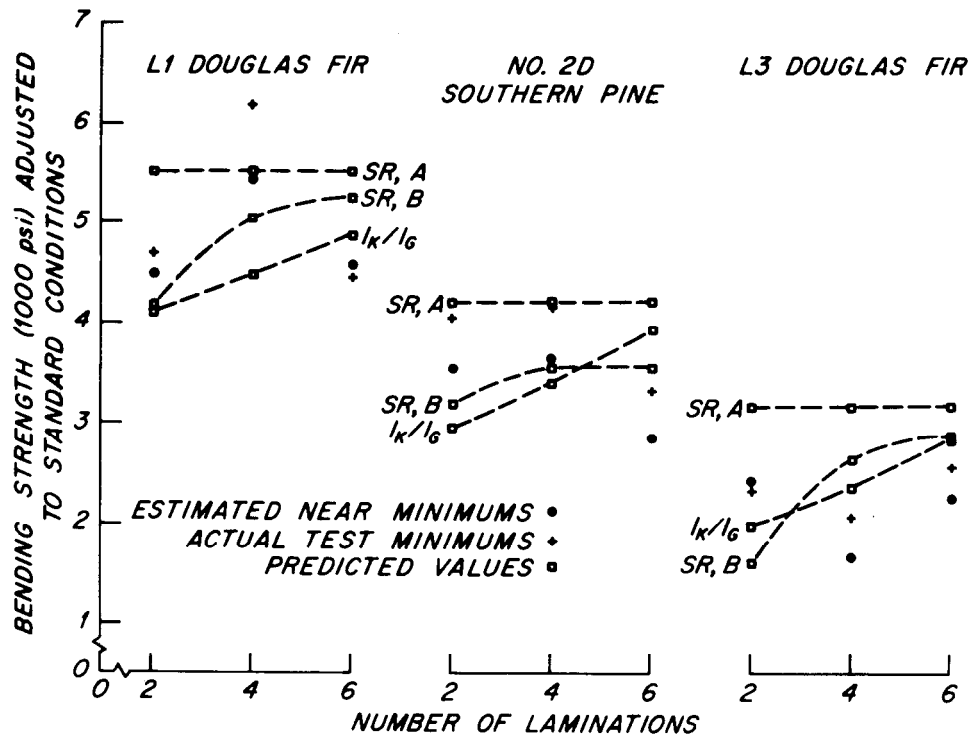


Figure 12.—Comparison of the results of this study with predicted values. The predicted values shown for the three methods are equal to the predicted design stress values in table 3 times 2.1. (M 148 628)

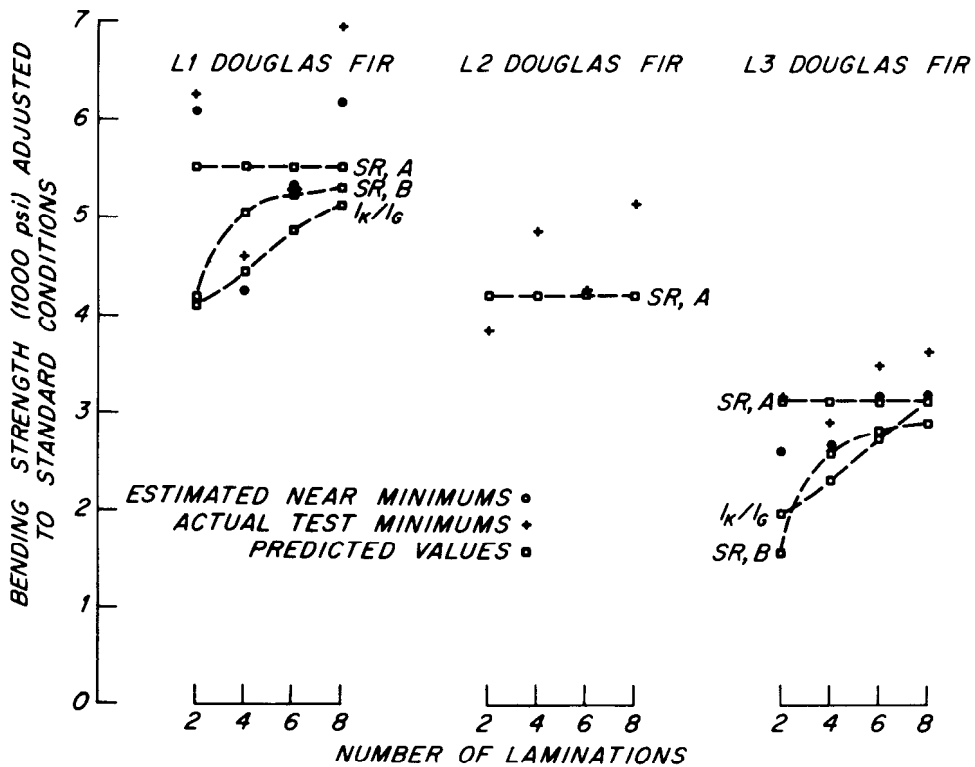


Figure 13.—Comparison of Johnson's results with predicted values. The predicted values shown for the three methods are equal to the predicted design stress values in table 3 times 2.1. (M 148 629)

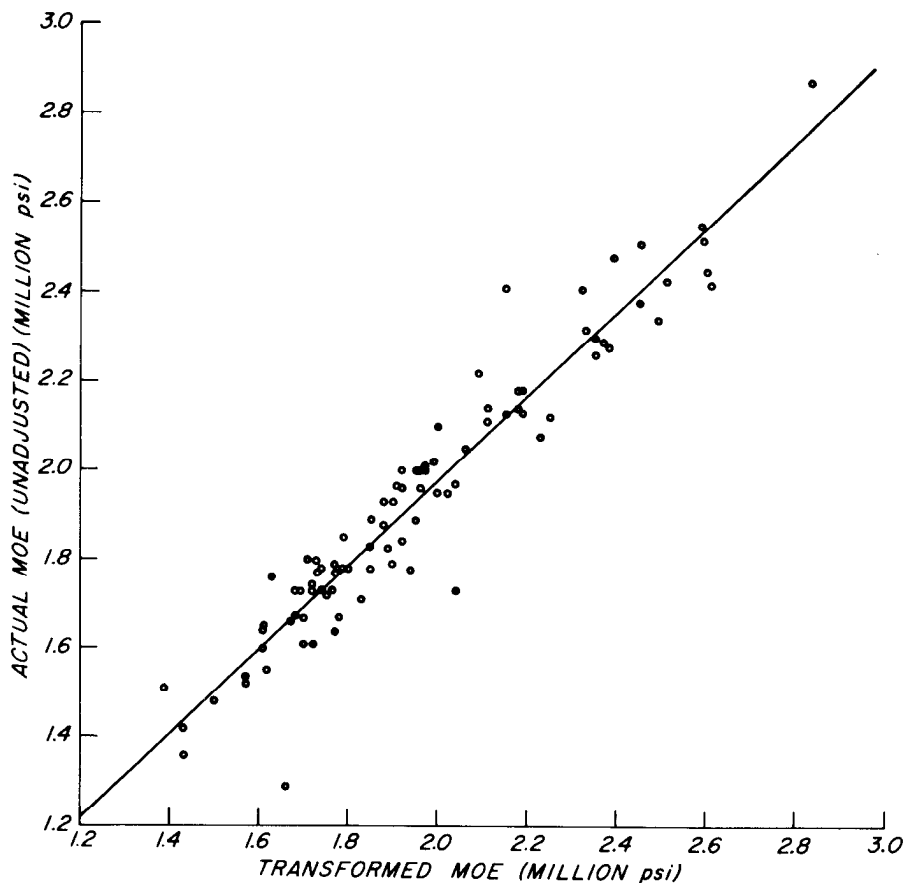


Figure 14.—Comparison of actual and transformed MOE values.

(M 148 630)

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APPENDIX A

Johnson's Data

J. W. Johnson conducted static bending tests of 2-, 4-, 6-, and 8-lamination beams at Oregon State University prior to 1969. Although he never published the results, he has given the authors permission to use his data; Table 7 summarizes those test results.

Johnson tested coast region Douglas-fir beams with 1-1/2-inch-thick laminations with knots occupying approximately 0.1, 0.2, 0.3, 0.4, or 0.5 of the cross section. The knots were placed above one another in the same 1-foot-long cross section which was located between the two load

points. The beams were tested under two-point loading, with the 36-inch distance between the load heads remaining constant for all tests. The total span between the reactions varied from 84 to 120 to 162 to 204 inches for the 2-, 4-, 6-, and 8-lamination beams, respectively, which resulted in shear span-to-depth ratios of 16:1, 14:1, 14:1, and 14:1, respectively.

A total of 100 shallow beams were tested with five beams in each of five quality and four size categories. All the beams failed at or near the critical cross section. The moisture content of the beams averaged about 10 percent.

The first step in analyzing Johnson's data was to make adjustments to standard conditions (3,

7) by applying the values given in table 8. After completing an analysis of variance on Johnson's data and on the combination of data from Johnson's study and this study (see appendix C), most calculations were made assuming the 0.1 and 0.2 material to be L1 (dense) and the 0.4 and 0.5 material to be L3 (medium grain). In addition, the 0.3 material was assumed to be L2 (medium grain). Near-minimum bending strength values were estimated in the same manner as this study's and are shown in table 5.

When Johnson's data were normalized and combined with this study's data to develop a new prediction method, the selected knot size for each group of beams was used.

Table 7.—Summary of Johnson's test results¹

Approximate knot size	Number of laminations	Modulus of rupture					Modulus of elasticity		
		Unadjusted		Adjusted to standard conditions ²		Coefficient of variation	Unadjusted	Adjusted ³	Coefficient variation
		Mean	Range	Mean	Range		Mean	Mean	
		Lb/in. ²	Lb/in. ²	Lb/in. ²	Lb/in. ²		Million lb/in. ²	Million lb/in. ²	
0.1	2	10,040	8,760-11,290	8,720	7,610- 9,800	9.3	2.03	1.96	7.5
.1	4	8,010	6,620- 9,540	7,250	5,990- 8,630	13.2	2.04	1.97	13.9
.1	6	8,980	5,680-11,950	8,340	5,270-11,090	25.8	2.36	2.29	8.3
.1	8	9,660	7,920-10,990	9,140	7,490-10,390	12.7	2.39	2.32	6.6
.2	2	8,860	7,220-10,820	7,700	6,270- 9,400	15.2	2.11	2.04	10.1
.2	4	7,120	5,090- 9,480	6,450	4,610- 8,580	27.1	1.89	1.83	11.8
.2	6	8,550	7,590- 9,830	7,940	7,050- 9,120	9.6	2.18	2.11	6.9
.2	8	8,110	7,350- 9,470	7,670	6,950- 8,960	10.2	2.22	2.15	1.6
.3	2	8,060	4,430-10,560	7,000	3,850- 9,170	32.3	2.00	1.93	11.7
.3	4	6,390	5,390- 7,500	5,790	4,880- 6,790	11.8	1.85	1.79	12.2
.3	6	6,580	4,540- 8,310	6,110	4,210- 7,710	22.1	1.96	1.90	9.2
.3	8	6,630	5,410- 9,040	6,270	5,120- 8,550	25.2	2.13	2.06	6.9
.4	2	4,700	4,040- 5,290	4,080	3,510- 4,590	9.5	1.71	1.65	5.1
.4	4	5,690	4,730- 6,900	5,150	4,280- 6,240	13.9	1.80	1.75	9.7
.4	6	5,650	5,130- 5,880	5,250	4,760- 5,460	5.4	1.84	1.78	6.1
.4	8	5,300	4,700- 6,670	5,020	4,450- 6,310	14.7	2.00	1.93	7.4
.5	2	4,810	3,670- 5,140	4,230	3,190- 4,700	32.9	1.62	1.57	9.3
.5	4	4,510	3,260- 6,740	4,080	3,000- 6,100	29.6	1.62	1.57	7.3
.5	6	4,360	3,700- 5,390	4,050	3,500- 5,000	14.6	1.84	1.78	12.3
.5	8	4,550	3,850- 6,060	4,310	3,640- 5,730	19.5	1.81	1.75	3.1

¹ Each tabulated mean is the result of 5 tested beams.

² Adjusted to standard conditions of 12 pct moisture content (3) and a 12-in.-deep beam, uniformly loaded and with a 21:1 span-to-depth ratio (7).

³ Adjusted to a moisture content of 12 pct (3).

APPENDIX B

Factors Applied to Test Results

Table 8.—Moisture content and size adjustment factors

Type of adjustment	Number of laminations	Data source	Modulus of rupture	Modulus of elasticity
Moisture content ¹	2,4,6	This study	2	2
	2,4,6,8	Johnson's study	0.953	0.968
Size ³ (7)	2	This study	1.079	None
4		1.048	None	
6		1.035	None	
2	Johnson's study	1.098	None	
4		1.053	None	
6		1.027	None	
8		1.008	None	

¹ Johnson's beams averaged a moisture content of approximately 10 pct. As more precise values for each beam are unknown, all the beams were adjusted from 10 to 12 pct moisture content by using the same factor (3).

² Equations used are from ASTM D 2915 (3).

³ Includes adjustment for depth, span-to-depth ratio, and method of loading.

Table 9.—Statistical factors used to estimate the fifth percentile with 75 percent confidence¹

Sample size	K	Sample size	K
5	2.463	29	1.873
9	2.141	30	1.869
10	2.103	31	1.864
11	2.073	40	1.834
15	1.991	60	1.795
20	1.933	190	1.725

¹ From table A-7 of (14).

APPENDIX C

Analysis of Variance

MOR and MOE Data

An analysis of variance (11) was conducted on the data from this study, Johnson's study, and the two studies' data combined. The results are listed in table 10. "Yes" indicates that the source of variation had a significant effect on MOR or MOE with 95 percent probability.

The analysis of the data from this study showed that grade/species had a significant effect on MOR and MOE, but the number of laminations or the interaction of the two did not.

The analysis of variance using Johnson's data gave identical results for MOR. However, a further breakdown of his MOR data showed that there was no significant difference between his beams containing 0.1 and 0.2 sized knots which approximate an L1 grade of Douglas-fir, and similarly for those beams containing 0.4 and 0.5 sized knots which approximate an L3 grade of Douglas-fir. There was also found to be no significant

difference between the 0.2 and 0.3 size knots.

The analysis of variance for Johnson's MOE values gave some unexpected results. It was discovered that his 2- and 4-lamination beams were significantly less stiff than his 6- and 8-lamination beams. The reason for this is unknown.

Combining Johnson's and this study's data and performing an analysis of variance showed a significant effect due to the study, grade, and interaction of the study, grade, and number of laminations. A closer examination of the data, however, showed that there was no significant difference between the L1 Douglas-fir material from this study and Johnson's study, or the L3 Douglas-fir material from the two sources. It was felt, therefore, that the L1 data from the two sources could be combined with little possibility of a study error; the same applies to the L3 data from the two sources.

Normalized MOR Data

A two-way analysis of variance package that could handle some empty cells (this study did not include any

8-lamination data) was conducted with the 190 normalized MOR data. Some problems were encountered with the No. 2D southern pine data because of the different maximum allowable edge and centerline knot sizes. The analysis was conducted three times, each time with a different SR value for the No. 2D data; the other 160 values were not changed. The No. 2D SR values were based on the maximum allowable edge knot size, centerline knot size, or the average of the two and were 0.66, 0.48, and 0.57 for 2 by 6 lumber, respectively. A grade effect was detected when the maximum centerline knot was used. Use of the maximum edge knot resulted in a grade effect at the 0.10 significant probability level, but not the 0.05 level. No effect was detected at either level when the average of the edge and centerline knots was used. Thus, this average predicted the SR for No. 2D southern pine that was most consistent with the other data.

Table 10.—Summary of variance results analysis¹

Source of variation	Dependent variable					
	This study		Johnson's study		This study and Johnson's study	
	Modulus of rupture	Modulus of elasticity	Modulus of rupture	Modulus of elasticity	Modulus of rupture	Modulus of elasticity
Grade ²	Yes	Yes	Yes	Yes		
Number of laminations	No	No	No	Yes		
Grade x number of laminations	No	No	No	No		
Study ²					Yes	No
Grade ²					Yes	Yes
Number of laminations					No	Yes
Study x grade					No	No
Study x number of laminations					No	Yes
Grade x number of laminations					No	Yes
Study x grade x number of laminations					Yes	No

¹ "Yes" indicates that the source of variation had a significant effect on the dependent variable with 95 pct probability

² Grade refers to L1, L2, or L3 Douglas-fir or No. 2D southern pine.

U.S. Forest Products Laboratory.

Bending strength of shallow glued-laminated beams of a uniform grade, by Catherine M. Marx and Russell C. Moody.

21 p. (USDA For. Serv. Res. Pap. FPL 380).

Develops a new method for determining appropriate design stresses for shallow beams. Also determines which of three present design criteria best predict near-minimum bending strength values for such beams.
